Mass Transfer of O_3 in Ultrapure Water through a boundary layer to the Wafer.

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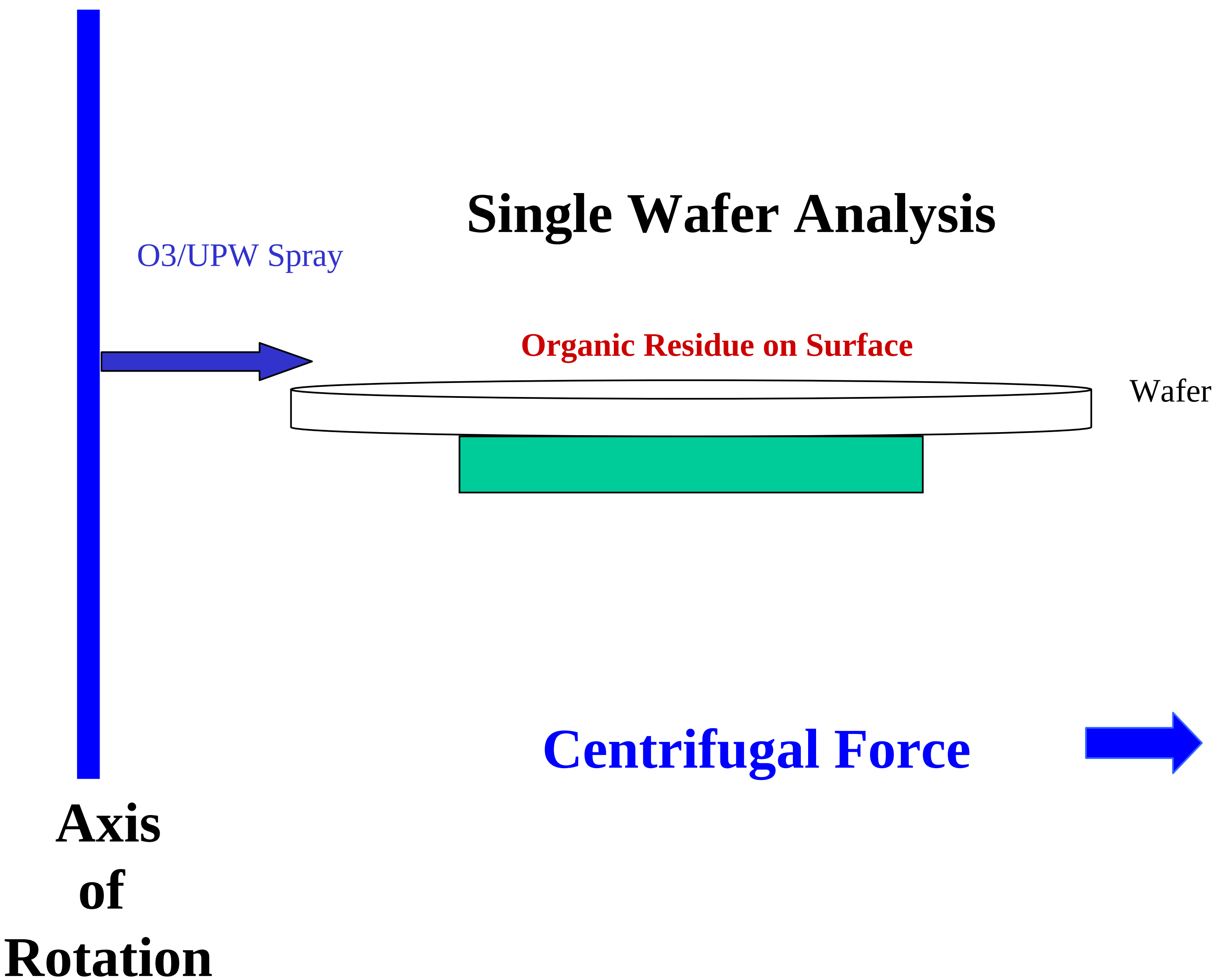
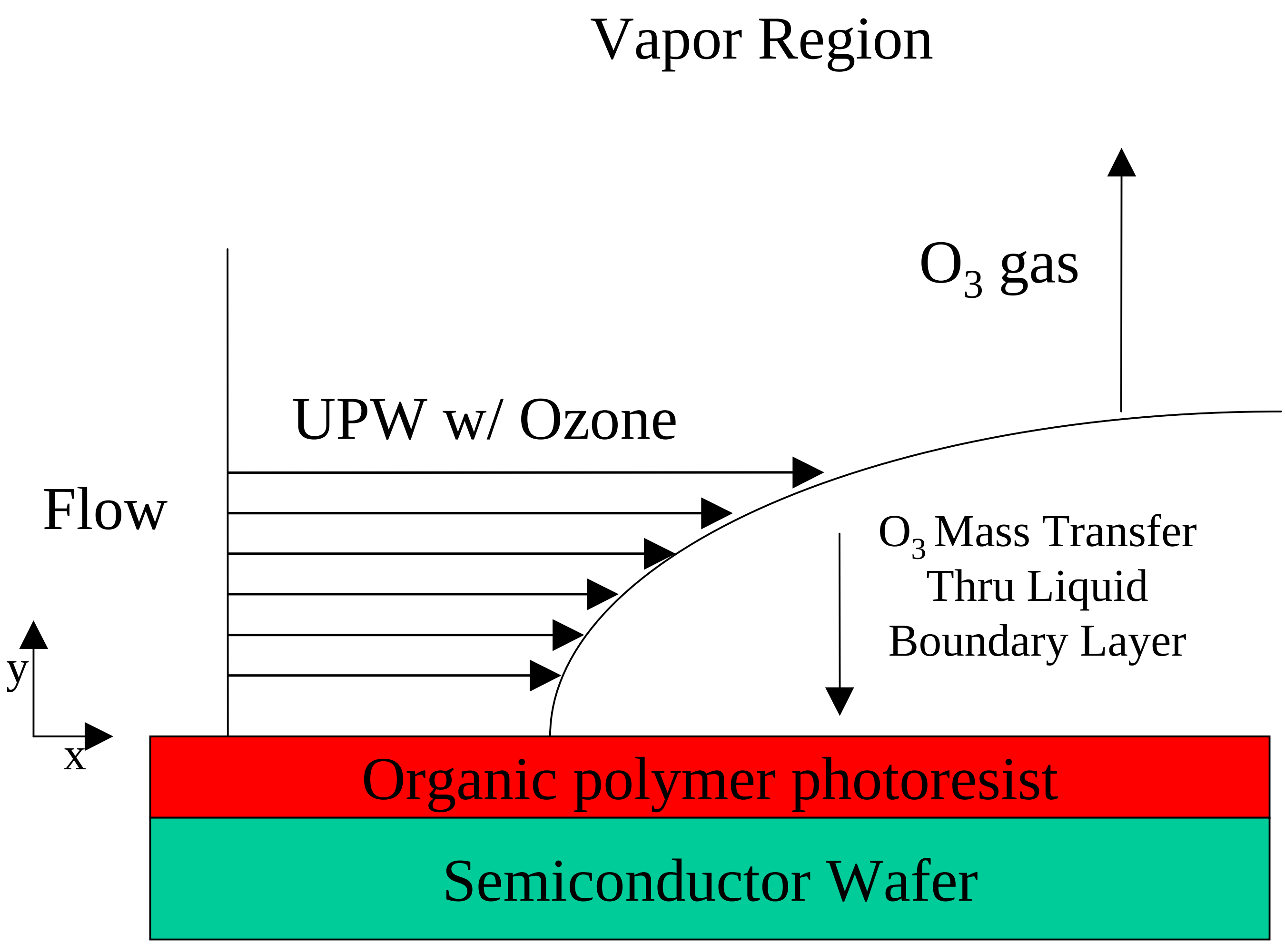
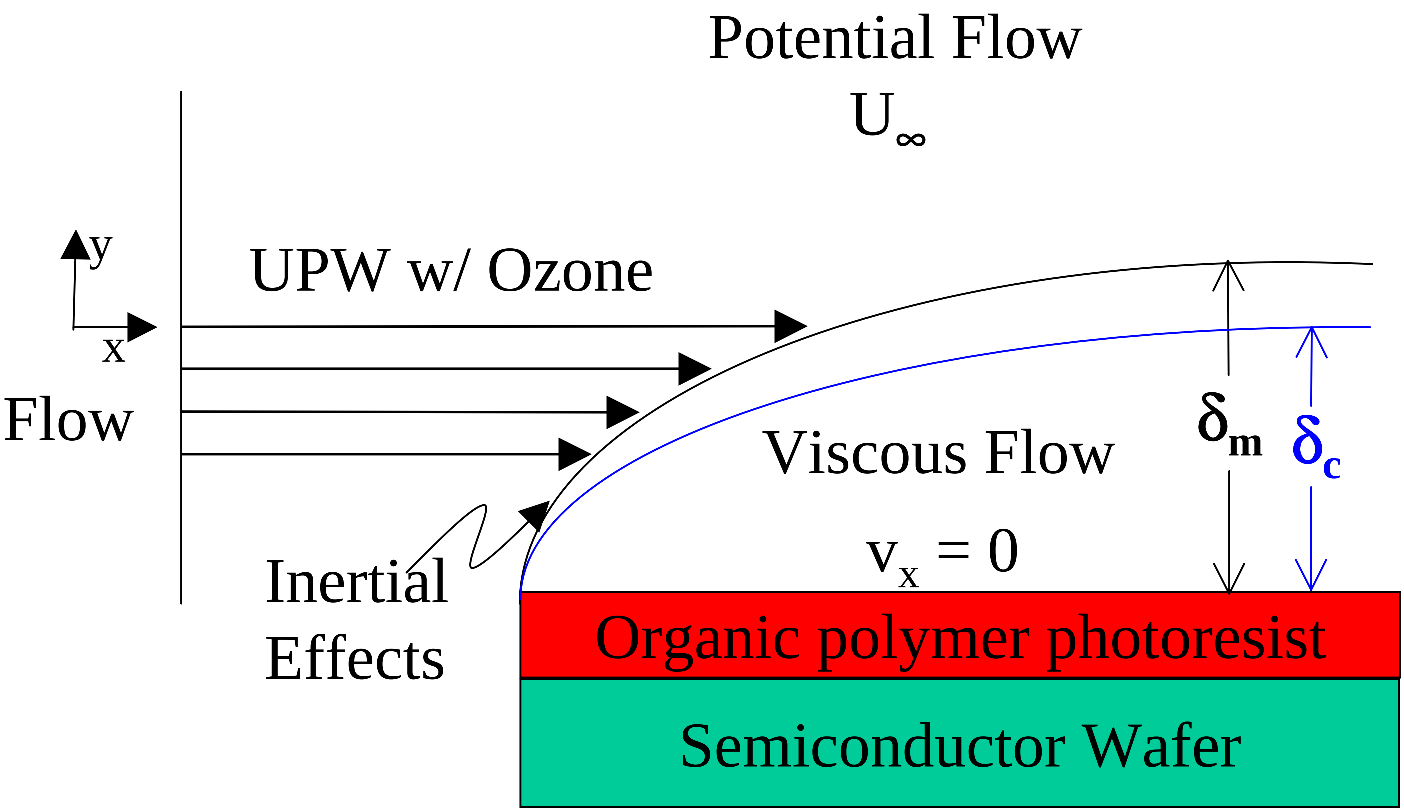
Single Wafer Analysis



Boundary Layer Theory





$$\tau_{yx} = -\mu \frac{\delta v_x}{\delta y}$$

Boundary Conditions:

@ $x = 0$; $v_x = U_\infty$; for all $y > 0$

@ $y = 0$; $v_x = v_y = 0$; no slip condition

@ $y = \infty$; $v_x = U_\infty$, $v_y = 0$; a flat plate scenario

For Ultrapure Water, $Sc \cong 1000 >> 1$.

Mass Transfer Equations

Continuity Equation:

$$\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y} = 0$$

Conservation of Momentum:

X momentum;

$$\rho\left(\frac{\delta v_x}{\delta t} + v_x\frac{\delta v_x}{\delta x} + v_y\frac{\delta v_x}{\delta y}\right) = \frac{-dP}{dx} + \mu\left(\frac{\delta^2 v_x}{\delta x^2} + \frac{\delta^2 v_x}{\delta y^2}\right) + \rho g$$

Y momentum;

$$\rho\left(\frac{\delta v_y}{\delta t} + v_x\frac{\delta v_y}{\delta x} + v_y\frac{\delta v_y}{\delta y}\right) = \frac{-dP}{dy} + \mu\left(\frac{\delta^2 v_y}{\delta x^2} + \frac{\delta^2 v_y}{\delta y^2}\right) + \rho g$$

Conservation of Mass:

$$\frac{\delta C_{O_3}}{\delta t} + v_x\frac{\delta C_{O_3}}{\delta x} + v_y\frac{\delta C_{O_3}}{\delta y} = D_{O_3/H_2O}\left(\frac{\delta^2 C_{O_3}}{\delta x^2} + \frac{\delta^2 C_{O_3}}{\delta y^2}\right)$$

These equations can be solved for the Flux of Ozone through the boundary layer, by determining the concentration boundary layer thickness δ_c. This can be determined from a scale analysis relative to the momentum boundary layer thickness δ_m .

A scale analysis performed on the x momentum equation leads to;

$$\frac{\delta_m}{x} = \frac{1}{(Re)^{1/2}}$$

For steady state conditions, with an incompressible fluid and a steady stream, the x momentum equation can also be simplified to;

$$v_x \frac{\delta v_x}{\delta x} + v_y \frac{\delta v_x}{\delta y} = \nu \frac{\delta^2 v_x}{\delta y^2}$$

Combining this Equation of Motion with the Continuity Equation, for $y << \delta_m$, using the combination of variables technique, these equations can be solved to yield;

$$\mathbf{v_x \cong \frac{U_\infty\, y}{\delta_m}}$$

$$\mathbf{v_y \cong \frac{\nu\, y^2}{\delta_m{}^2}}$$ **Very near the plate.**

The Conservation of Mass equation simplifies to;

$$v_x \frac{\delta C_{O_3}}{\delta x} + v_y \frac{\delta C_{O_3}}{\delta y} = D_{O_3/H_2O} \frac{\delta^2 C_{O_3}}{\delta y^2}$$

The concentration boundary layer thickness, δ_c, can be estimated from a scale analysis with the momentum boundary layer thickness, δ_m.

This leads to;

$$\delta_c = \frac{\delta_m}{(Sc)^{1/3}}$$

$$\delta_c = \frac{x}{(Re)^{1/2}\,(Sc)^{1/3}}$$

Flux of Ozone : $$N_{O_3} = \frac{-D_{O_3/H_2O}}{\delta_c}\, C_{O_3}$$